On the Luminosity Curves of Persons having Normal and Abnormal Colour Vision.

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§ 1. When reading a recent paper before the Royal Society, and also in the Press, Dr. Edridge Green has stated that he can find no connection between the luminosity and the colour sense of persons having either normal or abnormal colour sensations. Since I feel that to allow such a statement to go unchallenged might be interpreted as meaning that no such connection could be shown to exist, I propose in the following paper to place before the Society some of the evidence which indicates that there is in reality a very intimate relation between luminosity and colour sense. The results given include a small part of those which have been obtained in a series of experiments which have occupied the last two years and form part of an investigation which is still in progress.

The term "luminosity" as used in this paper has the following meaning: Suppose that light from some source, such as the electric arc, is admitted to a spectroscope by means of which a real pure spectrum is produced, and that a slide in the plane in which the spectrum is formed carries a slit of fixed width. Light of sensibly one wave-length, *i.e.* monochromatic light, will pass through this slit, and by means of a lens placed in the beam of this light an image of the first face of the prism which is used to form the spectrum can be formed on a screen. In this way a monochromatic patch of light is obtained, the brightness of which depends on the nature of the source of light, the width of the collimator slit, the width of the slit placed in the spectrum, which for short will be called the movable slit, and the dimensions of the lenses employed. Further, if alongside this coloured patch is formed a white patch of light produced by light which proceeds from the same source but has not undergone dispersion, and that by some means or other the intensity* of this white light is altered till the coloured and white patches appear to the eye equally bright, then the intensity of the white light, measured in any arbitrary units, measures the luminosity of the light of that colour which is passing through the movable slit. Since the unit in which the white light is measured is arbitrary, we are not concerned with the absolute intensity of illumination of the white patch, and may use any device we like to alter the quantity of white light which falls on the screen so long as we are able to measure the ratio of the quantity of light employed in different experiments. It will further be observed that for any given person the measurement of the luminosity of a given coloured light in the spectrum involves the comparison of the brightness of the coloured patch as it appears to him with the brightness of the white patch as it appears to him.

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* The physical brightness of a light, *i.e.* the stimulus, will be spoken of as the ntensity, the term brightness being reserved for the sensation produced when the light nters the eye.

The above definition of what is meant by the luminosity of a colour in the spectrum of a given source is equivalent to that employed by Sir William Abney in all his work, and the only reason for giving it is that to follow the reasoning used in this paper it is essential that this definition should be kept in mind. A convenient arrangement for conducting measurements of luminosity has been described by Abney and Festing,* and has been employed in making the observations given below.

This is hardly the place to discuss the accuracy with which we are able to judge of the comparative brightness of, say, a white and a green light. It is admittedly a difficult operation, but the results which are quoted below show that if the conditions are suitably chosen the measurement can be performed with considerable consistency. The difficulty of comparing the brightness of a coloured light with that of a white light is almost completely obviated if a flicker method is employed, in which the coloured and white lights are made to successively occupy the same spot on the screen and the intensity of the white is altered till the flicker vanishes. If suitable precautions are taken even quite inexperienced observers are found to be capable of making quite consistent measurements by the flicker method, and a satisfactory piece of apparatus for such measurements is described later.

If the movable slit described above is placed in different parts of the spectrum and the intensity of the white which appears equally bright is measured in each place, on plotting these intensities as ordinates, the abscissæ being prism deviations, a luminosity curve is obtained, that is a curve which gives the luminosity of the different colours in the spectrum of the source and for the particular prism employed.

As has been mentioned, the unit in which the intensity of the white is measured is quite arbitrary, and throughout this paper it is taken as such that for a normal eye the maximum luminosity in the spectrum of the brightness employed is 100.† The abscissæ of the luminosity curves given are also expressed in arbitrary units which are proportional to the deviations produced by the prism system, and for convenience of reference to Sir William Abney's published measurements the unit employed is the same that he uses. The connection between these standard scale readings, which will be indicated by the letters S.S.N., and the wave-lengths are given in Table I. It would be possible to reduce the results so as to make them correspond to a normal spectrum, *i.e.* a grating spectrum, when they would be independent of the dispersion curve of the particular prism employed. Since, however, we shall only be dealing with the differences of the luminosity curves obtained by different persons it seems hardly necessary to go through the somewhat laborious calculations involved.

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* 'Phil. Trans.,' 1886, Part II, p. 455.

† The intensity of illumination of the screen corresponding to 100 is throughout about 5 lux, or 0·5 candle-feet.

Table I.

Scale number.	Wave-length.	Scale number.	Wave-length.
S.S.N.	Å.U.	S.S.N.	Å.U.
62	6957	44	5481
60	6728	42	5373
58	6521	40	5270
56	6330	38	5172
54	6152	36	5085
52	5996	34	5002
50	5850	32	4924
48	5720	30	4848
46	5596	28	4776

Suppose that when making a set of observations we start with the movable slit at the extreme red end of the spectrum and determine the intensity w_1 of the white which appears of the same brightness as the colour and then move the slit towards the blue through a distance equal to its width and again determine the intensity w_2 of the white, and so on throughout the spectrum. We should in this way determine piece by piece the brightness of the whole spectrum, and the sum $w_1+w_2+w_3+$, etc., could be taken to represent the total brightness of the whole spectrum. If now the slide carrying the slit were removed, so that the light corresponding to the whole spectrum was allowed to fall on the same portion of the screen, thus forming white, and we now determine the intensity W of the comparison white which is equal in brightness to this recombined spectrum. Then it has been shown by Abney, Tufts, Ives and others* that

$$W = w_1+w_2+w_3+, \text{ etc.}$$

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* This statement requires a certain amount of limitation, and must be held only to apply to spectra of moderate brightness and to sources of light such that the light is not chiefly due to the blue and violet. The proposition that the luminosity of the sum of two lights is equal to the sum of the separate luminosities does not apply to very bright lights, and in the case of very small intensities may not apply in the blue and violet.

That is, the luminosity of the recombined spectrum is equal to the sum of the luminosities of its parts.

Now the sum $w_1 + w_2 + w_3 +$, etc., is proportional to the area enclosed by the luminosity curve. Hence the area of the luminosity curve represents the total brightness of the light which is formed into the spectrum and is, therefore, a constant whatever the condition of the vision of the person who makes the observations. This is at once apparent if we remember that if the brightness of the whole recombined spectrum is compared with the comparison white, since these whites are derived from the same source and must, therefore, have exactly the same composition, a setting which appears correct to one person must also appear correct to any other, whatever the differences which may exist between their vision.

The fact that with a given spectroscope and source of light, if the scale used for measuring the white is the same for all measurements, the area of the luminosity curve will be always the same is of great utility, since it allows us to correct for any variation in the unit in terms of which the white is measured, such as may occur owing to the mirrors, lenses, etc., becoming tarnished. It may be mentioned here that when a colour-blind observer makes a luminosity-curve determination I have often obtained my own curve at the same time. The two sets of measurements being made immediately one after the other there is no time for the instrument to alter, and the areas of the curves obtained have always been the same, although the shapes of the curves are often entirely different.

§ 2. *Calculation of the Form of the Luminosity Curve for Colour-Deficient Observers.*—Before giving the luminosity curves obtained in the case of colour-deficient observers, it will be convenient to calculate what would be the form of the curves on certain assumptions. We shall then be in a position to judge of the accuracy of these assumptions by comparing the calculated curves with those obtained by observation.

Starting from the well-established fact that all colours can be matched by a combination, in suitable proportions, of three properly selected colours, Thomas Young and Helmholtz developed a theory of colour vision which supposes that the machinery by which colour is perceived consists of three sets of mechanism, and that the sensation produced by light of any colour is the sum of the sensations due to the individual mechanisms. Or, as we may say, the sensation produced is the sum of the separate sensations produced by these separate mechanisms.

It is hardly necessary here to consider the line of reasoning by which the amount of each sensation produced by the different colours of the spectrum can be derived from experiments in which a coloured light of one wave-length is matched by the mixture of lights of two other wave-lengths. Suffice it to say that Koenig and Abney have published curves which give for each wave-length the values of the three sensations. These three sensations are generally referred to as the red, green, and blue sensations. These names are rather unfortunate, as persons unfamiliar with the subject are apt to suppose that the green sensation, say, is alone operative when green light enters the eye. As a matter of fact, in the case of a normal eye, all three sensations are affected by green light. The extreme red is the only part of the spectrum which affects but one of the mechanisms.

Given the luminosity curve for any given source of light, and making use of Abney's values* for the ratio of the ordinates of the three sensation curves for the different spectral colours, we can construct three sensation curves such that for any wave-length the sum of the ordinates of the three curves is equal to the ordinate of the luminosity curve. In this way we obtain the sensation curves in terms of luminosity, and the thin line curves given in fig. 1 are Abney's curves for the crater of the electric arc, the thicker line curve being the corresponding luminosity curve.

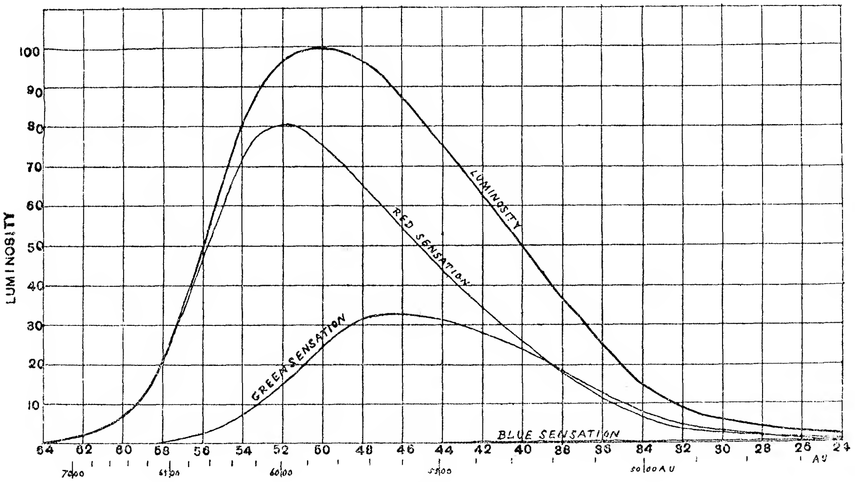

Fig. 1.

When white light, that is light of all wave-lengths, enters the eye, the effect produced on the three receptive mechanisms will be proportional to the areas of the three sensation curves. The relative areas of these curves for the electric arc are:—

Area of red sensation curve	579
Area of green sensation curve	248
Area of blue sensation curve.........	3·26

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* 'Phil. Trans.,' 1905, A, vol. 205, p. 333.

We have now to consider the manner in which the sensation curves of a person who has defective colour vision differ from the normal. In several papers, Abney has brought forward evidence that, at any rate, in the case of the more ordinary forms of colour defect, either the red sensation curve or the green sensation curve is modified, and that this modification is such that the shape of the curve remains unaltered, each ordinate being reduced in the same proportion.* Thus, if the red sensation is deficient, and, say, the ordinate of the red sensation curve for any given wave-length is half the corresponding ordinate for the normal, then for all other wave-lengths the ordinates of the red sensation curve are also half those of the normal red sensation curve. Hence it follows that the area of the red sensation curve must also be half the area of the corresponding curve for the normal.

If we possessed any method of measuring in absolute units the sensation produced when light of a given intensity enters the eye, we should be able to tackle the question as to whether the maximum ordinate of the sensation curves was the same for all persons. A certain amount of evidence on this matter has been obtained by Abney by the study of the minimum intensity of light required for vision. For the present, however, we are confining ourselves to a study of the relative sensations produced by given amounts of light of different colours for each observer separately. Thus we take some one kind of light as a standard and compare the relative stimulation of the sensations produced by other kinds of light with that produced by this standard light. This is what is done when determining a luminosity curve, the particular light chosen as a standard being white light, *i.e.* the light which we are going to use to produce the spectrum which supplies the different colours which are to be used in the measurements.

Having to use white light as a standard, when we come to compare the measures made by a person having defective colour vision with those made by a normal, the matter is complicated by the fact that the unit with which the comparison is made, that is the white, is not the same for both. The defective sensation will not only affect the sensation received from the coloured light, but also that received from the white. Abney's theory as to the way the sensation curves of persons having defective colour vision differ from the normal, however, enables us to get over this difficulty, as will be shown below.

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* Another way in which we might have a departure from the normal would be if, say, the green sensation curve were displaced in such a way that its maximum occurred at a different wave-length. There seems evidence that such a displacement may occasionally be met with, and the author hopes shortly to discuss this question in another communication. He has calculated the luminosity curves corresponding to such displacement and finds that they all intersect at neighbouring points, which do not coincide with the point P referred to on p. 411.

As long as we confine our attention to the part of the spectrum between the extreme red and the blue, the effect of the blue sensation on the luminosity may in general be neglected as being too small to produce any appreciable effect*; hence, in what follows, the effect of the red and green sensations on the luminosity will alone be considered.

The easiest way to show the manner in which the luminosity curve of a colour-deficient person is obtained will be to consider a particular case, say, one where the red sensation is deficient to such an extent that all the ordinates of the red sensation curve are only half the normal. Such a person may be said to possess half-normal red sensation and will be indicated by the symbol 0·5 R.S. Since each of the ordinates of the red sensation curve is half the normal, the total area of that curve will also be half the normal. As in the light from the crater of the electric arc the areas of the red and green sensation curves are as 579 to 248, it follows that the areas for the 0·5 R.S. are as 290 to 248.

Now suppose such a person determines the luminosity of a colour of which the wave-length is λ, and that λ_r and λ_g are the ordinates of the normal red and green sensation curves for this colour, the corresponding ordinates for the observer will be $\frac{1}{2}\lambda_r$ and λ_g. The total sensation produced by the colour will be the sum of the two sensations, that is, for the normal it will be $\lambda_r + \lambda_g$ and for the observer (0·5 R.S.) $\frac{1}{2}\lambda_r + \lambda_g$.

The sensation produced by the comparison white in the luminosity measurement will be proportional to the sum of the areas of the red and green sensation curves. Hence, if we represent the areas of these curves for the normal by Σr and Σg respectively, the sensations produced by the white for the normal will be $\Sigma r + \Sigma g$, and for the colour-deficient will be $\frac{1}{2}\Sigma r + \Sigma g$.

Thus the brightness of the coloured light is for the 0·5 R.S. observer reduced in the ratio $(\frac{1}{2}\lambda_r + \lambda_g)/(\lambda_r + \lambda_g)$, while that of the white is reduced in the ratio $(\frac{1}{2}\Sigma r + \Sigma g)/(\Sigma r + \Sigma g)$. Let w be the intensity of the white when the normal observer makes the luminosity setting and w' the intensity of the white when the colour-deficient observer makes the setting. Then we have

$$w = a\,\{\lambda_r + \lambda_g) \tag{1}$$

and

$$w' \frac{\{\frac{1}{2}\Sigma r + \Sigma g\}}{\Sigma r + \Sigma g} = a\,\{\tfrac{1}{2}\lambda_r + \lambda_g\}, \tag{2}$$

where a is a constant which depends on the unit used to measure the intensity of the white light.

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* In the case of a normal eye the total luminosity due to the blue sensation when white light enters the eye is only about 1/200 of that due to the red sensation.

Thus the ratio of the colour-deficient observer's luminosity to that of the normal is given by

$$\frac{w'}{w} = \frac{\frac{1}{2}\lambda_r + \lambda_g}{\lambda_r + \lambda_g} \times \frac{\Sigma r + \Sigma g}{\frac{1}{2}\Sigma r + \Sigma g}. \tag{3}$$

Thus, knowing w, we can calculate what is the ordinate w' of the luminosity curve for the 0·5 R.S. observer corresponding to the colour λ.

The following statement may make the above argument clearer. When a colour-deficient observer makes a luminosity setting he matches the brightness of the colour as it appears to him against the brightness of the white as this appears to him. Owing to his deficiency both the colour and the white appear less bright. If there were no reduction in the brightness of the white to him, his luminosity setting would be $(\frac{1}{2}\lambda_r + \lambda_g)/(\lambda_r + \lambda_g)$ of the normal setting. Owing, however, to the comparison white being also reduced in brightness the size of his white unit is reduced in the ratio of $\frac{1}{2}\Sigma r + \Sigma g$ to $\Sigma r + \Sigma g$; and hence, as the size of the unit has decreased, he requires more of these units to match the colour, that is, we must multiply the number given above by $(\Sigma r + \Sigma g)/(\frac{1}{2}\Sigma r + \Sigma g)$. This decrease in the value of the white unit is the reason why the luminosity curve for a red-blind observer is higher than the normal in the green, and that for a green-blind observer it is higher than the normal in the red. These high values do not indicate that a red-blind person receives a greater stimulus from a given green light, or a green-blind receives a greater stimulus from a red light, than does a normal, but simply that relatively to the stimulus received from a given white light the stimulus received is greater.

Using equation (3) and the luminosity curves for a normal eye given by Abney, the luminosity curves for persons having the following red sensations in terms of the normal, namely, 0 R.S., 0·33 R.S., 0·7 R.S., and the following green sensations 0 G.S., 0·33 G.S., and 0·6 G.S., have been calculated. The results together with the normal luminosity curve are shown in fig. 2.

It will be observed that all the luminosity curves intersect at one point, P, which corresponds to S.S.N. 48·8 or a wave-length 5770 Å.U. The condition that the luminosity should be the same for the normal as for, say, the 0·5 R.S. observer is that [equations (1) and (2)]

$$\{\tfrac{1}{2}\lambda_r + \lambda_g\} \frac{\Sigma r + \Sigma g}{\frac{1}{2}\Sigma r + \Sigma g} = \lambda_r + \lambda_g,$$

or

$$\frac{\lambda_r \Sigma g}{2} = \frac{\lambda_g \Sigma r}{2}, \quad \text{or} \quad \frac{\lambda_r}{\lambda_g} = \frac{\Sigma r}{\Sigma g}, \tag{4}$$

that is at a wave-length such that for both observers the ratio of the ordinates of their red and green sensation curves is the same as the ratio of the total areas of these curves.* Equation (4) shows that the wave-length of the light corresponding to the point of intersection is independent of the amount or kind of the deficiency in colour sensation of the observer.

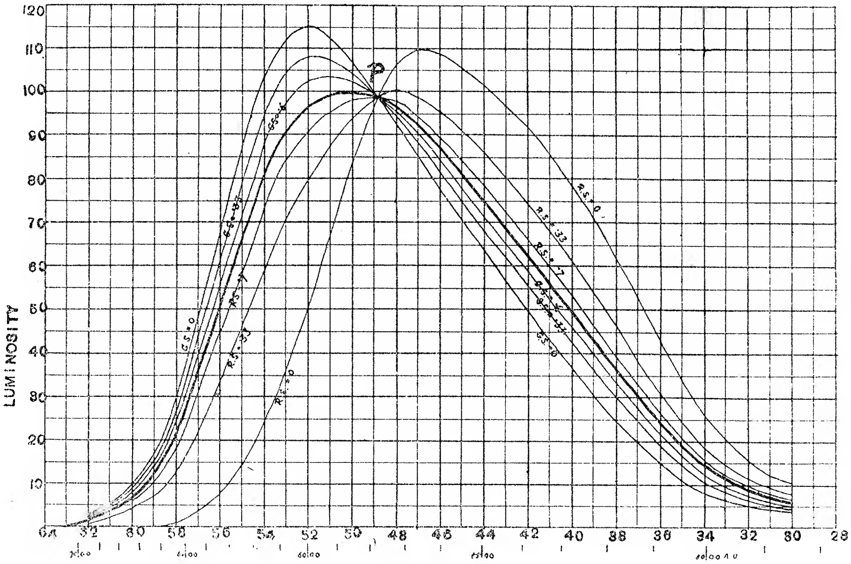

FIG. 2.

The curves given in fig. 2 depend on—

(1) The accuracy of Abney's sensation curves.

(2) The correctness of Abney's theory that in the case of the ordinary types of total or partial red or green colour-blindness the ordinates of one of the sensation curves are all reduced in the same proportion, and

(3) The additive property which has been assumed and which involves the corollary that the areas of the luminosity curves obtained by normal and colour-deficient persons are the same. Thus if it can be shown that the observed luminosity curves of persons who are colour deficient agree with the calculated curves it is strong evidence in favour of the correctness of the above three assumptions.

To test this point the observed luminosities obtained by colour-deficient observers will be shown plotted on diagrams containing the calculated curves shown in fig. 2, so that it will be possible to see at a glance whether (1) the observed points all lie on one of these curves or lie uniformly between two adjacent curves indicating that the observer belongs to a class intermediate between those for which the curves are plotted, and (2) whether the luminosity is the same for all for one particular colour.

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* The wave-length at which all the curves will intersect depends on the distribution of light in the spectrum employed in the experiments, *i.e.* on the source of light.

Two sets of data will be used to test the validity of the calculated curves. In the first place Sir William Abney has published in various papers the results of a large number of measurements of luminosity for colour-deficient persons. These measurements extend over many years and the numbers given do not always directly enable us to test the question, since the scales on which they are given vary; in many cases the maximum luminosity for each observer is taken as 100, but other scales for the ordinates are also employed. However, by making use of the fact that if the curves were all plotted on the same scale then the areas must all be the same (the source of light and the spectroscope were the same throughout), the factors by which the values given must be multiplied so as to make the areas equal have been calculated. When these factors have been applied the curves will all be on the same scale. In each individual case the same factor of course is used for all the observations. The readings reduced in this manner to the same scale are those given in the figures. These data make a peculiarly valuable test because the numbers used were all published before the calculated curves, which we are going to use them to test, were obtained and in fact some of them were published* before the sensation curves used in the calculations had been obtained. It is also open to anyone to check the results by reference to the original papers.

Secondly, use is made of a selection from a large number of fresh measurements obtained in a different manner. Since the equality of brightness method of obtaining the luminosity curves described above, involving as it does the comparison of the brightness of two lights which differ greatly in colour, is a process which requires considerable practice, although given such practice Abney's numbers show that concordant measures can be obtained, the flicker method originally described by Ferry,† has been utilised for the determination of luminosity curves. In one arrangement which has been found to give good results, even with quite inexperienced observers, a cylinder of large diameter rotating about a horizontal axis is placed between the movable slit and the screen on which the light is received. Segments are cut out of the circumference of the cylinder so that the screen is alternately illuminated by the coloured light and the comparison white, both kinds of light falling on the same part of the screen. In order to obtain satisfactory measurements it is essential that the alternations from colour to white and *vice versâ* should take place without overlap or gap between the lights. If either of these occur then complete absence of flicker can only be obtained when the speed of flicker is high, and under these conditions the sensitiveness of the method is too low to admit of satisfactory measurements being obtained.

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* Sir W. Abney has stated in one of his papers that the numbers given in the papers which are quoted were all obtained before he had determined the sensation curves.

† 'Amer. Journ. Sci.,' 1892, vol. 44, p. 192.

Since with a flicker method it is impossible to alter the intensity of the white by means of rotating sectors, for if this is done a phenomenon analogous to beats is obtained, use was made of an annulus of gradually increasing density placed in the white beam. This annulus had been calibrated by Sir William Abney, and is described in one of his papers.*

Another form of flicker apparatus† has also been used, in which the source of light is a Nernst filament and in which the alternations are caused by rotating a disc between the filament and the collimator lens. Half of the circumference of this disc is cut away and the other half forms a mirror from which the white light used for the comparison light is reflected. Thus, during half the time of rotation of the disc the light passes through to the spectroscope, and during the other half it is reflected to form the comparison light. In this instrument the intensity of the white light is altered by altering the relative positions of two systems of lenses which are placed in the path of the white light.

In the flicker experiments the light was received on a block of compressed magnesium carbonate, of which the surface had the form of a square, each edge being 18 mm. This surface was at a distance of 60 cm. from the eye, so that each side of the square subtended an angle of 1° 43′ at the observer's eye.

The luminosity curves obtained by the flicker method differ slightly from those obtained by the equality of brightness method, particularly in the green and blue. These differences have been the subject of a separate investigation carried on by Sir W. Abney and the author, and they hope to be able to communicate the results shortly.

Since, at any rate to a first approximation, the additive law applies to flicker measurements,‡ without considering the differences between the luminosity curves obtained by the two methods, we may by a similar line of reasoning to that used above calculate from the normal flicker luminosity curve the flicker luminosity curves for the colour-deficient, and then by

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[*] 'Phil. Trans.,' 1897, A, vol. 190, p. 156.

[†] The cost of which has been partly defrayed by a grant from the Government Grant Committee.

[‡] Tufts, 'Physical Review,' 1907, vol. 25, p. 438; Whitman, 'Physical Review,' 1895, vol. 3, p. 248.

comparing these calculated curves with the observed curves we can test the accuracy of the calculated curves and consequently the correctness of the assumptions on which they are based.

§ 3. Taking first the measurements previously published, the observed points are shown plotted in figs. 3 to 7. To save space the points corresponding to two observers are plotted on the same figure, while the references to the paper in which the original measurements are published are given below each figure.*

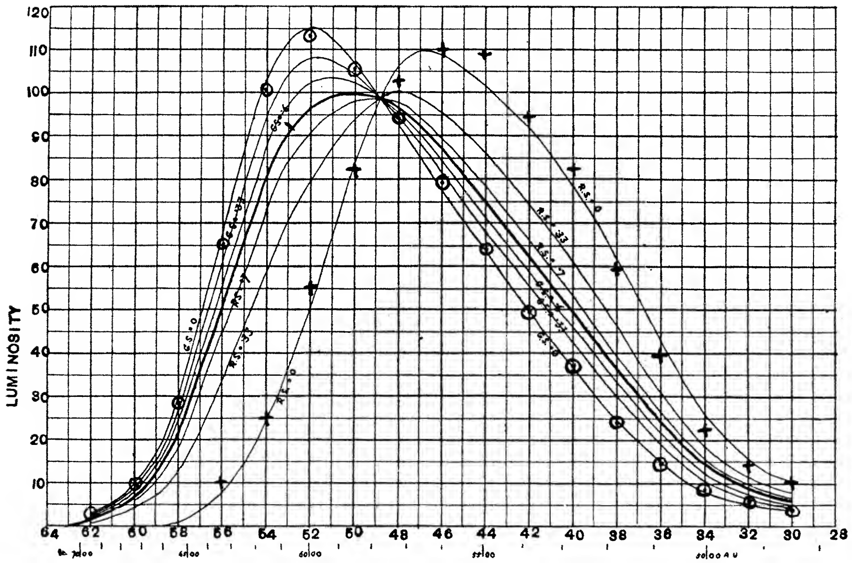

Fig. 3.

+ L.
⊙ F. } ‘Roy. Soc. Proc.,’ 1910, A, vol. 83, p. 472.

In figs. 3 and 4 we have examples of persons who have R.S. = 0 and G.S. = 0 respectively. In fig. 5 observer D has G.S. = 0 and observer Z has R.S. = 0·33. In figs. 6 and 7 we have two examples of persons having R.S. = 0 and one having G.S. = 0·2.†

It will be noticed that when the observed points are plotted on such a scale that the area of the curve drawn through them is the same as the

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* All the numbers given in this paper were obtained when the observer was dark adapted.

† On plotting the numbers given in the ‘Philosophical Transactions’ for V. H., it was obvious that some error had crept in, and on submitting the matter to Sir W. Abney, he said he had already noticed that, owing to a mistake when reducing the scale readings of the instrument to S.S.N., the numbers as given were erroneous. He has kindly looked up the original measurements and the points given in fig. 6 are obtained from these numbers.

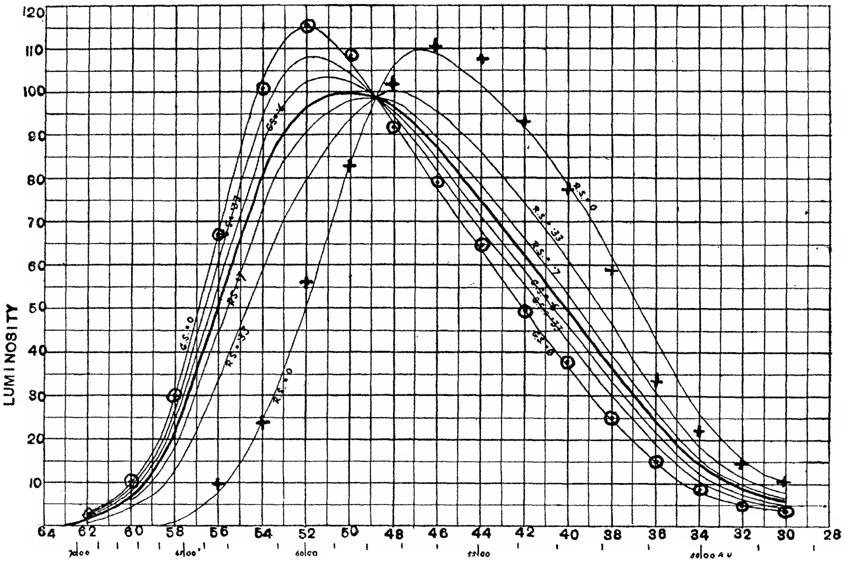

FIG. 4.

+ H.
⊙ E. } 'Roy. Soc. Proc.,' 1910, A, vol. 83, p. 472.

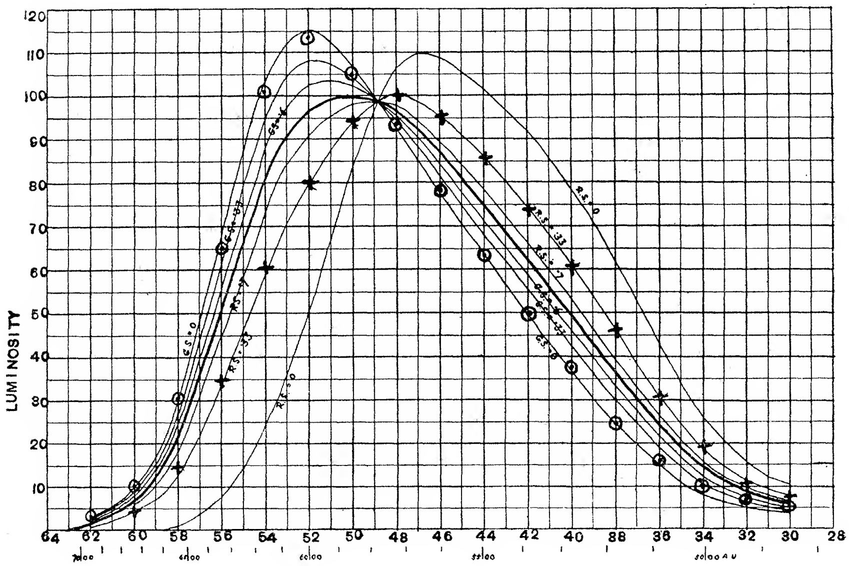

FIG. 5.

+ Z., 'Roy. Soc. Proc.,' 1910, A, vol. 83, p. 460.

⊙ D., 'Roy. Soc. Proc.,' 1910, A, vol. 83, p. 472.

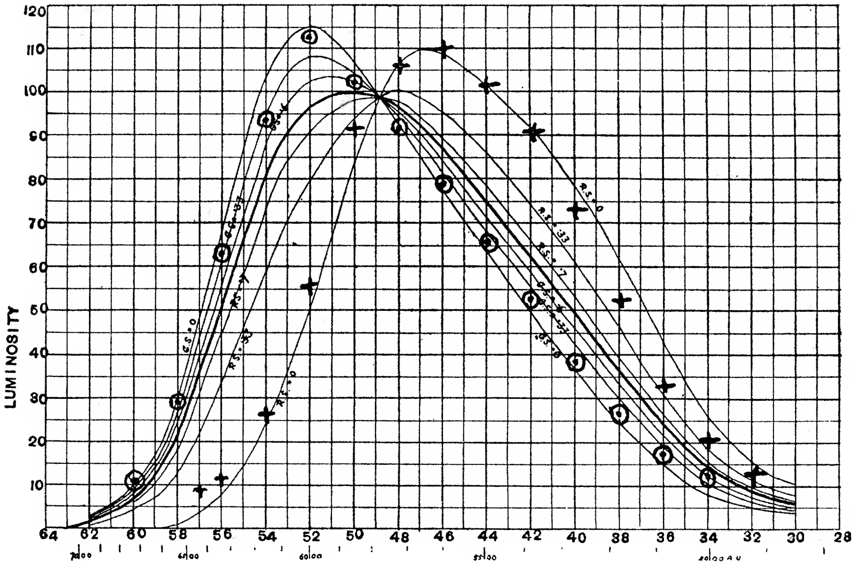

FIG. 6.

+ H. R., 'Phil. Trans.,' 1892, p. 556.

⊙ V. H., 'Phil. Trans.,' 1892, p. 555.

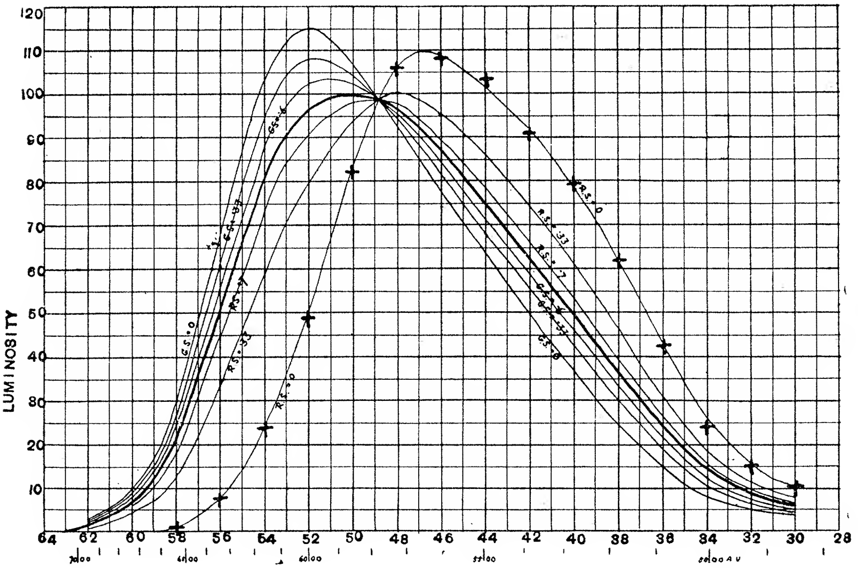

FIG. 7.

+ X., 'Roy. Soc. Proc.,' 1910, A, vol. 83, p. 466.

area of the normal curve, all the luminosity curves intersect at a single point, that is, for this particular point of the spectrum the luminosity is the same for all. Further, the plotted points agree quite well with the calculated curves. Any departure which is found is mostly confined to the greenish-blue, where, as has been already mentioned, differences may be expected, since, in the first place, when obtaining the calculated curves, we have neglected the effect of the blue sensation, and further (as will be shown in § 5), slight differences in macular pigmentation begin to be important in this region.

§ 4. A few examples of the results obtained by the flicker method are given in figs. 8, 9, and 10. The normal curve* is shown in each figure by the thick line. It will be observed that the normal curve differs from that obtained by the equality of brightness method given in fig. 2. This difference is partly due to a difference in the source of light employed. The flicker curves were obtained with an arc in which the positive carbon was horizontal, so that the crater is directly facing the slit of the spectroscope. The equality of brightness curves were obtained using an arc with nearly vertical carbons, so that the crater was turned more or less edgeways towards the slit. Most of the difference between the two curves is, however, a real difference, due to the difference of method. This difference is most marked in the green and

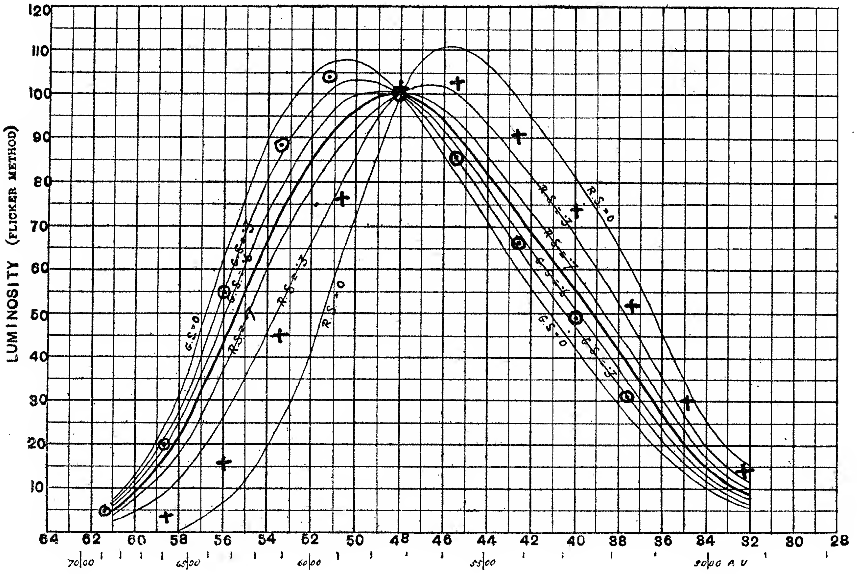

Fig. 8.

+ No. 1. ⊙ No. 2.

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* *May* 5, 1913.—This is the mean obtained by 10 observers.

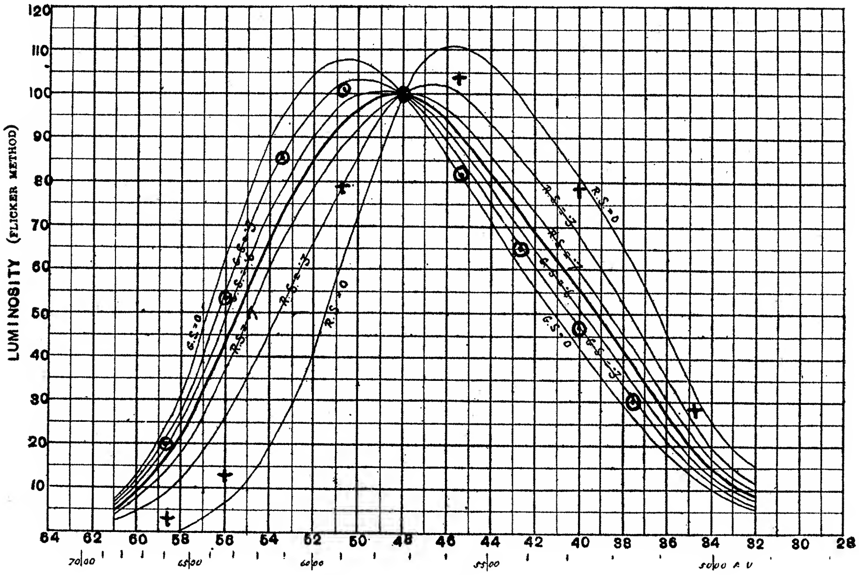

Fig. 9.

+ No. 4. ⊙ No. 3.

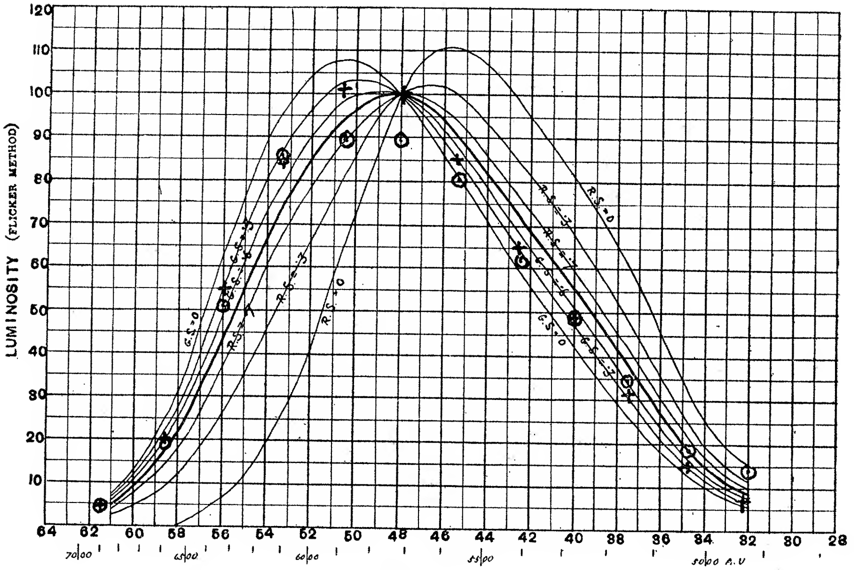

Fig. 10.

+ No. 5, foveal. ⊙ No. 5, parafoveal.

blue, and has been examined by Ives* in a series of most interesting and valuable papers. When comparing the results given in this paper with those given by Ives, it must be noted that Ives, in place of varying the intensity of the comparison white, varies the intensity of the coloured light. Hence, in his experiments, the brightness of all parts of the spectrum was equal when the comparisons with white were made, while, in my experiments, the brightness of the coloured light, and hence also of the comparison white, was very much less at the ends of the spectrum than at the middle.

In fig. 8, Observer 1 has about 0·2 R.S., while Observer 2 has about 0·3 G.S. In fig. 9, Observer 4 has about 0·2 R.S., and Observer 3, 0·3 G.S. In fig. 10, the points marked by a + show that Observer 5 has a little over 0·3 G.S.

Observer 1 has a shortened spectrum, a neutral point, and he mistakes red for green, and *vice versâ*. He matches a mixture of green and violet light with white, and any amount of deep red light could be added to the mixture without his being aware that any change had been made, although this addition of red caused the mixture, to a normal eye, to change from a bluish-green to a bright red. He is quite unable to correctly distinguish the colours of ships' lights.

Observer 2 has a neutral point, and calls the green of this part of the spectrum white. He often calls a green light white or a white light green, and, with lights of small intensity and small angular magnitude, he occasionally confuses green and red.

Observer 3 frequently calls white green and green white, and, with lights of small angular magnitude, he occasionally calls red green.

Observer 4 has a shortened spectrum and confuses red and green. With ships' lights, at distances up to 2 sea miles, he made over 20 per cent. of mistakes.

Observer 5 has a neutral point and calls green white and white green. If a little white light is added to any green, he calls the mixture white, although to a normal eye it still appears a fairly saturated green.

§ 5. In the preceding two sections examples have been given of what may be termed "normal" luminosity curves for persons having both normal and defective colour vision. The great majority of the luminosity curves obtained belong to this class. In a few cases, however, both of persons possessing normal colour vision and of those possessing defective colour vision, the luminosity curves differ in that they do not agree with any of the curves given in fig. 2, nor do they lie uniformly between any adjacent pair.

Taking first the case of abnormal curves given by persons who have normal colour vision. In figs. 11 and 12, the points marked by +, and through which the chain-dotted line is drawn, correspond to the luminosity measurements made by two such observers. In neither case do the curves pass through the point P, one curve passing considerably above this point and the

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* 'Phil. Mag.,' July, September, and December, 1912.

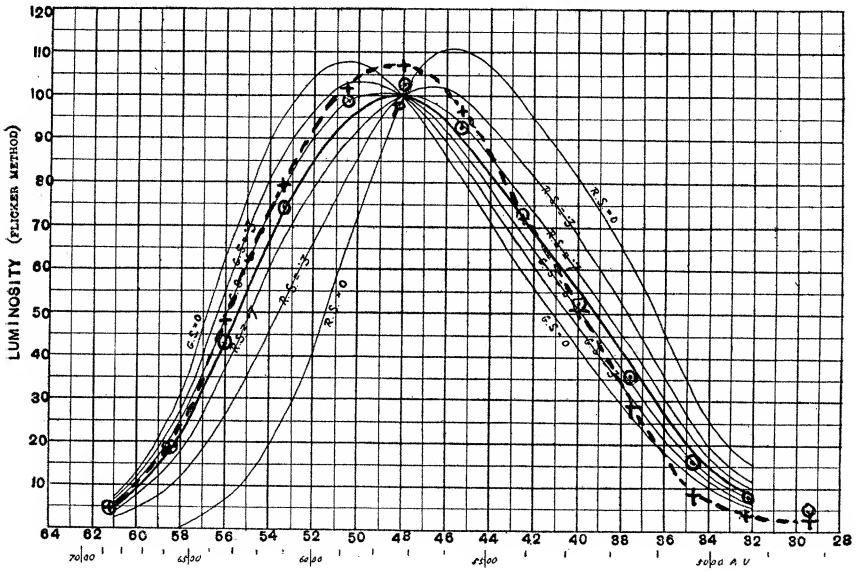

FIG. 11.
+ No. 6, foveal. ⊙ No. 6, parafoveal.

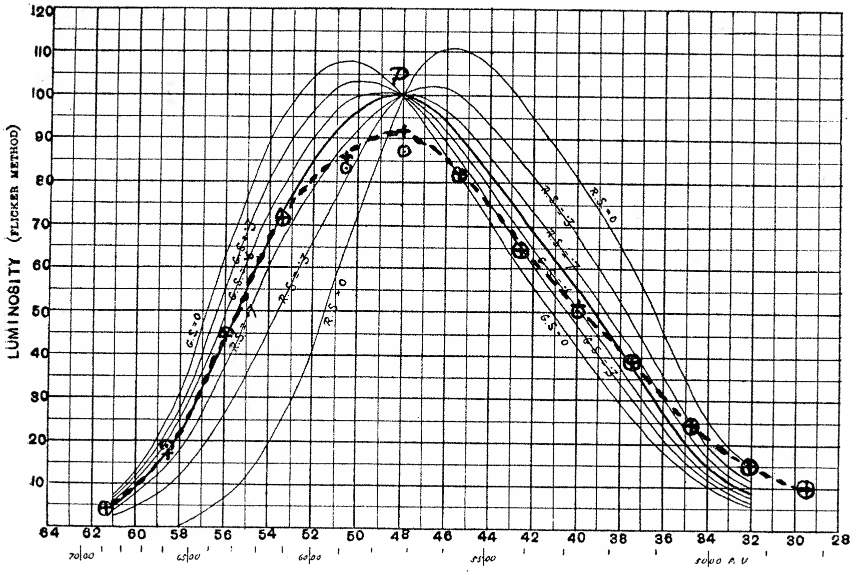

FIG. 12.
+ No. 7, foveal. ⊙ No. 7, parafoveal.

other below. The difference from the normal seems to be due to a cause to which Maxwell* called attention, namely, the pigmentation of the yellow spot. Thus it would appear that Observer 6 has more pigmentation at the fovea than the normal, and Observer 7 has less. In order to test the correctness of the above explanation, the luminosity curves of these observers, as well as of the author, were obtained when the image formed on the retina was confined to a region surrounding the fovea, that is to what, for short, may be called the "parafoveal" region. For this purpose the surface on which the colour and the white were received alternately had the form shown in fig. 13. It consisted of a white ring of which the diameter of the internal border subtended an angle of 3° 22′ at the eye of the observer, while the external border subtended an angle of 5°. A small white dot at the centre served to fixate the eye. Although this arrangement is more difficult to use than the small square, yet with a little practice fairly good settings could be obtained. The points marked ⊙ in figs. 11, 12, and 14 give the results obtained for the parafoveal region for Observers 6, 7, and the author.

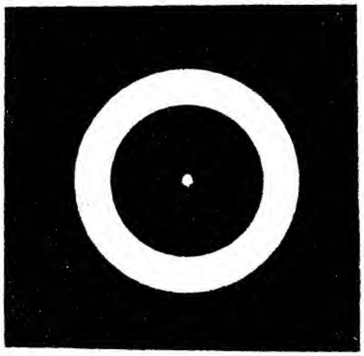

FIG. 13.

The first thing that strikes one is that in the case of Observer 7 the luminosity curve is practically the same for the parafoveal and the foveal regions, and that both of these agree with the author's parafoveal curve. Next, in the case of Observer 6, the parafoveal curve more nearly agrees with the foveal curve for the normal. This distinction is brought out very clearly in fig. 15, where the ratio of the parafoveal luminosity to the foveal for No. 7 and W. W. are shown. It will be seen that for Observer 7 the ratio is practically constant and equal to unity throughout the spectrum, while for W. W. the ratio increases rapidly as the blue end of the spectrum is reached. It would thus appear that Observer 7 has no more pigment at

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* 'Phil. Trans.,' 1860, vol. 150, p. 76.

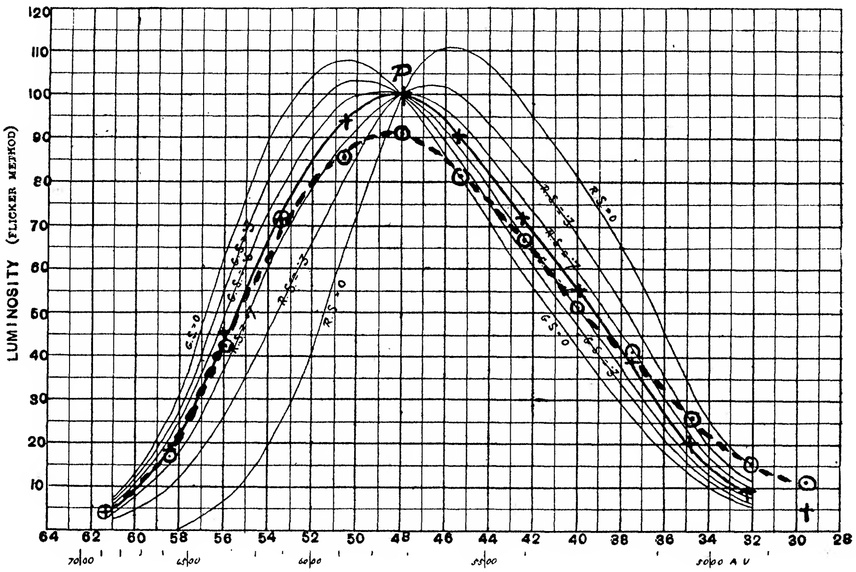

FIG. 14.

+ W. W., foveal. ⊙ W. W., parafoveal.

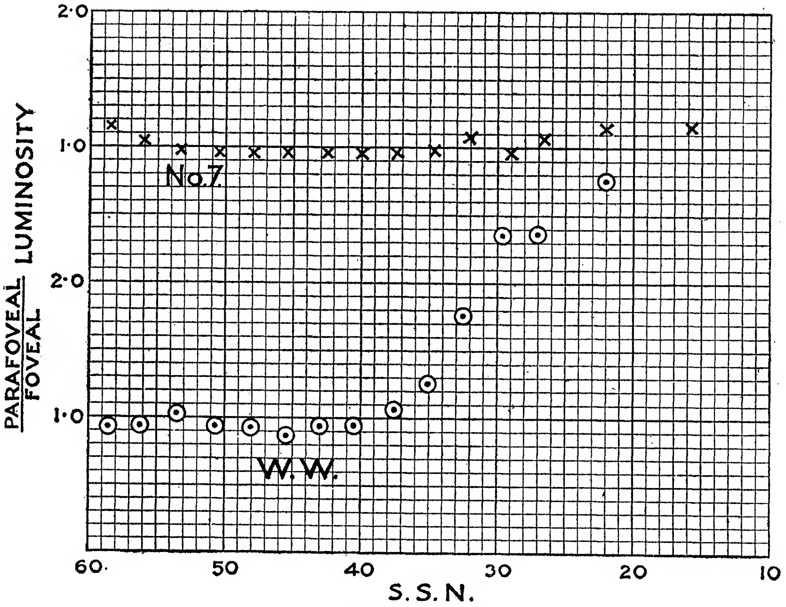

FIG. 15.

his fovea than the normal possesses in the parafoveal region. Further, the results support the view that Observer 6 has more than the normal amount of pigmentation at his fovea and also in the parafoveal regions. When Observer 7 made his settings, using spectacles fitted with light canary-coloured glass, his luminosity curve for foveal vision agreed almost exactly with the normal curve. In the case of Observer 7 the absence of the yellow pigment, which to the normal eye produces marked absorption in the blue and violet regions of the spectrum, enabled him to make accurate flicker settings far down into the violet, where to the normal eye the light is too feeble to allow of good settings being obtained. On the other hand Observer 6 cannot make reliable settings beyond the blue-green, the light appearing to him too feeble.

Observer 6 appears to have quite normal colour sense, although when matching a white by means of a mixture of red, green, and violet light he requires little or no violet, the reason being that owing to macular absorption the white is to him much "yellower" than to a normal eye, and hence the mixture of red and green which to the normal looks yellower than the white appears to him a correct match. He is quite good at matching blues and violets.

Observer 7 has also normal colour sense, although for his white match he uses more violet than the normal.

In figs. 16 and 17 are shown the flicker luminosity curves of two observers who have defective colour vision, the green sensation being in each case in defect, and who, in addition, appear to have more than the normal amount of pigmentation of the fovea. In fig. 18 Observer M* has more pigmentation than the normal, and his G.S. is practically zero. Observer N. W., on the other hand, is defective as to red sensation but has less pigmentation than the normal. It would be very interesting to examine the parafoveal luminosity curves for these colour-deficient observers, but unfortunately up to now it has been impossible to get them to devote the necessary time to make the observations.† The parafoveal curve for Observer 5 is indicated by the points marked ⊙ in fig. 10. The pigmentation of this observer appears to be normal.

In fig. 19 are given the luminosity curves of two persons who were

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* The values given in figs. 18, 19, 20 were all obtained by the equality of brightness method.

† A displacement of the green sensation curve towards the red end of the spectrum would give a luminosity curve very like those obtained by Observers 8 and 9. A study of their parafoveal luminosity curves might enable one to say whether they are cases of green deficiency plus abnormal macular pigmentation or cases of such displacement.

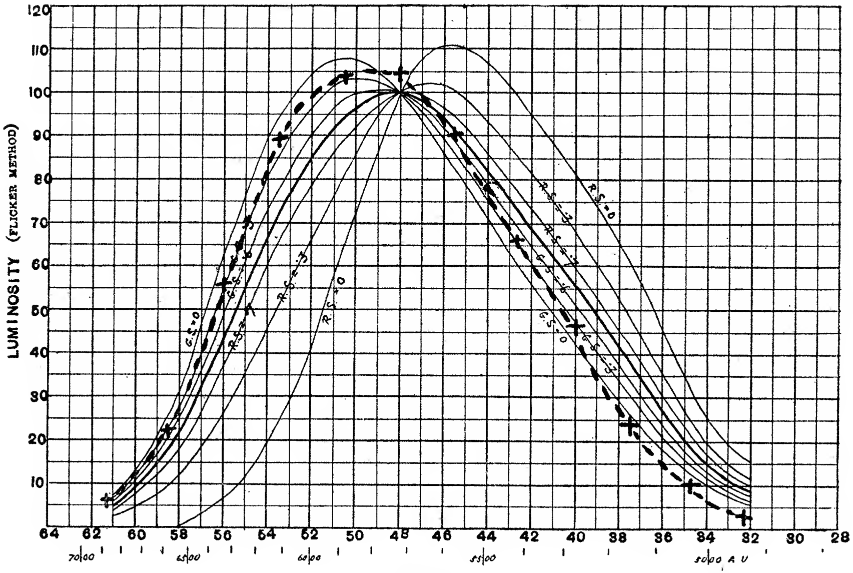

FIG. 16.
+ No. 8, foveal.

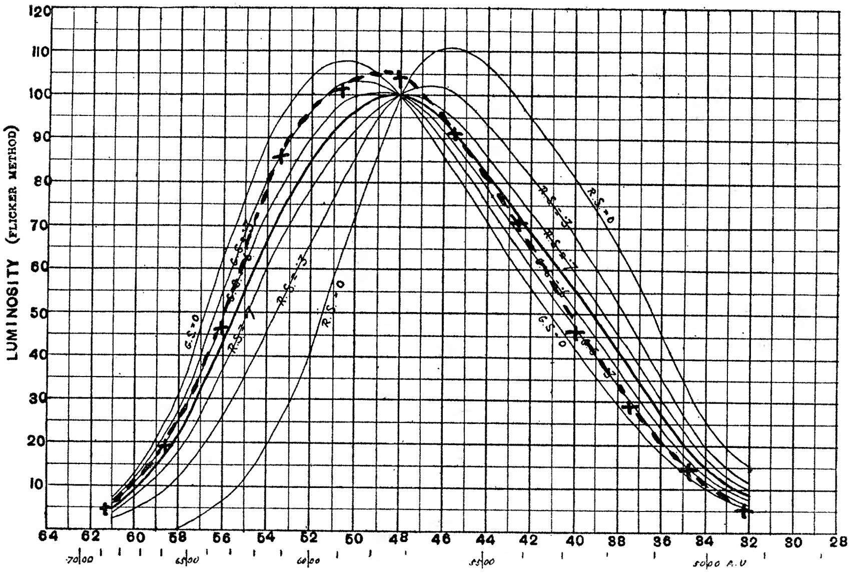

FIG. 17.
+ No. 9, foveal.

suffering from tobacco scotoma, and it will be observed that the curves are of an entirely different type to any of those previously given. In fig. 20

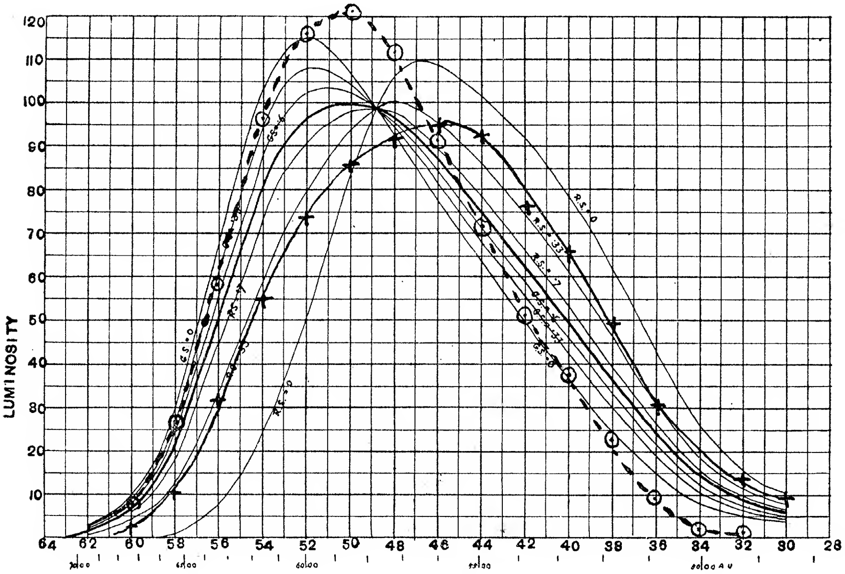

FIG. 18.

+ N. W. 'Roy. Soc. Proc.,' 1891, vol. 49, p. 503.
⊙ M., 'Phil. Trans.,' 1892, A, vol. 183, p. 551.

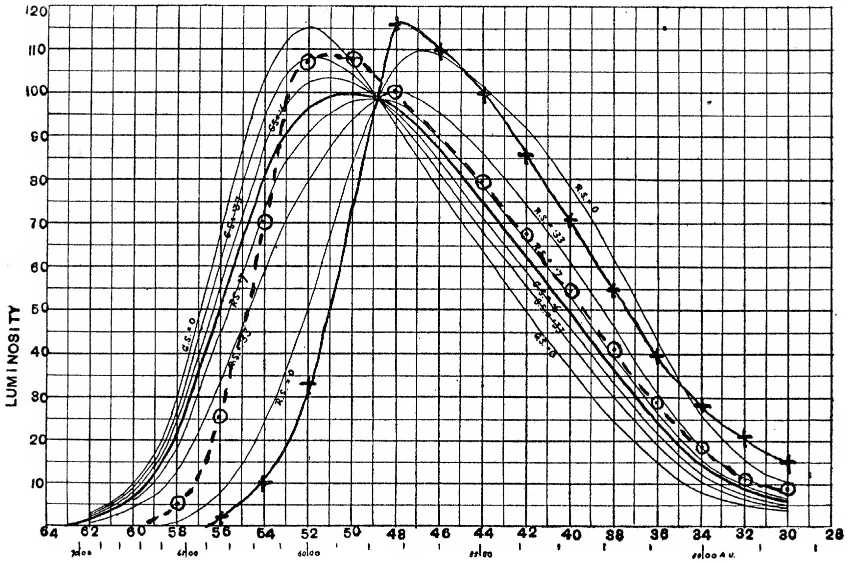

FIG. 19.

+ G., 'Roy. Soc. Proc.,' 1891, vol. 49, p. 502.
⊙ C., 'Roy. Soc. Proc.,' 1891, vol. 49, p. 493.

are given the curves for two observers who are colour blind, the defect being due to disease (atrophy of the optic nerve). In one case (Miss W.)* the curve very nearly agrees with the normal, while in the other the curve somewhat resembles that of Observer 7. It would thus appear that in the case of disease the colour-blindness which is produced is of an entirely different type to that which is found in cases of congenital colour-blindness. If this is the case comparison of the luminosity curves of the two classes of colour-blinds seems as if it might give us valuable information as to the seat of the colour-perceiving apparatus.

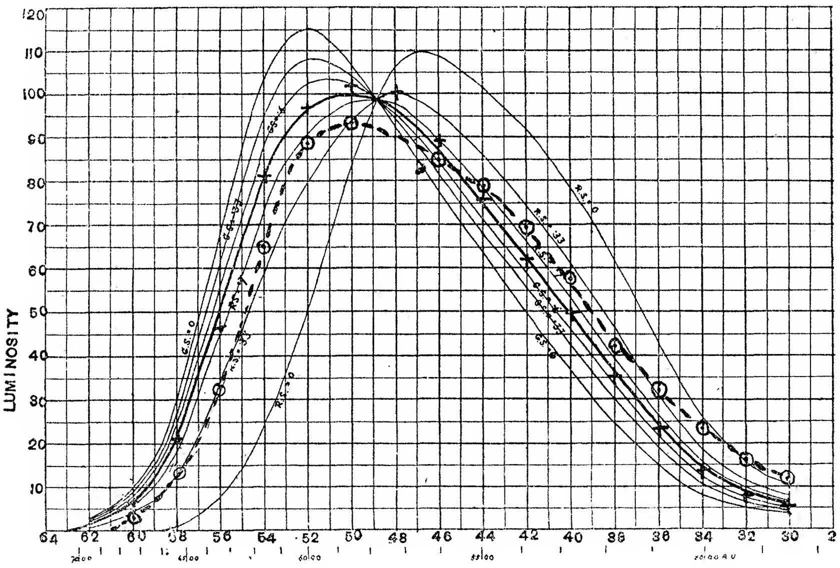

FIG. 20.

+ Miss W., 'Roy. Soc. Proc.,' 1891, vol. 49, p. 507.
⊙ W. S., 'Roy. Soc. Proc.,' 1891, vol. 49, p. 498.

The examples of the luminosity curves of different colour-deficient persons which have been given form only a small proportion of the cases which the author has examined. They will, however, be sufficient to show that the assumptions on which the calculated curves are based must, in the main, be true. Abney's theory that the different classes of the more commonly occurring forms of congenital colour defect are due to different degrees of deficiency in the red or green sensations is so strongly supported as to practically amount to a complete proof. It must be remembered that the investigation given above has been confined to that portion of the spectrum on the red side of the blue. If we wish to extend our investigation into the blue and violet then we should have to take account of the effect of the blue sensation. Further, although it seems quite clear that the major part of the luminosity perceived in that part of the spectrum including the red, yellow and green, at any rate with fairly bright spectra, is due to the red and green sensations, there is much evidence to indicate that, at any rate in the green and blue, there is a fourth sensation which may be called the fundamental white sensation. This probably corresponds to the rod-effect of von Kries, which would have to be taken into account if extremely minute differences of luminosity were under consideration, or if we were dealing with spectra of small intensity.

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* The eye tested had become "colour blind" after a stroke of paralysis; the other eye was still normal.

The Reflection of X-rays by Crystals.

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(Received April 7,—Read April 17, 1913.)

In a discussion of the Laue photographs it has been shown* that they may conveniently be interpreted as due to the reflection of X-rays in such planes within the crystal as are rich in atoms. This leads at once to the attempt to use cleavage planes as mirrors, and it has been found that mica gives a reflected pencil from its cleavage plane strong enough to make a visible impression on a photographic plate in a few minutes' exposure. It has also been observed that the reflected pencil can be detected by the ionisation method.†

For the purpose of examining more closely the reflection of X-rays in this manner we have used an apparatus resembling a spectrometer in form, an ionisation chamber taking the place of the telescope. The collimator is replaced by a lead block pierced by a hole which can be stopped down to slits of various widths. The revolving table in the centre carries the crystal. The ionisation chamber is tubular, 15 cm. long and 5 cm. in diameter. It can be rotated about the axis of the instrument, to which its own axis is perpendicular. It is filled with sulphur dioxide in order to increase the ionisation current: both air and methyl iodide have also been used occasionally to make sure that no special characteristics of the gas in

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* W. L. Bragg, 'Proc. Camb. Phil. Soc.,' vol. 17, Part I, p. 43.

† W. H. Bragg, 'Nature,' Jan. 23, 1913.